SYSTEM OF PARTICLES

11th Standard CBSE

Date : 27-Nov-22

Physics

Reg.No. :

Exam Time : 01:00:00 Hrs

Total Marks : 100

80 x 2 = 160

1) A person is standing on a rotating table with metal spheres in his hands. If he withdraw his hands to his chest, what will be the effect on his angular velocity?

2) Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius about an axis passing through their axis of symmetry?

3) Two boys of the same weight it at the opposite ends of a diameter of a rotating circular table. What happens to the speed of rotation if they move nearer to the axis of rotation?

4) If ice on poles melts, then what is the change in duration of day?

5) The speed of the whirl wind in a tornado is alarmingly high. Why?

6) Centre of gravity of a body on the earth coincides with its centre of mass for a small object and for a large object, it may not. What is the qualiative meaning of small and large in this regards? For which following two of them coincides, a building, a pond, a lake, a mountain.

7) A solid cylinder of mass 20 kg rotates about its axis with angular speed of 100 rad/s. The radius of cylinder is 0.25m. What is KE of rotation of cylinder?

8) If earth contract to half its radius. What would be the length of the day?

9) Explain how a cat is able to land on its feet after a fall taking the advantage of principle of conservation of angular momentum?

10) A horizontal disc rotating about a vertical axis perpendicular to its plane and passing through centre makes 180 rpm. A small lump of wet mud of mass 10 g falls on disc lightly and sticks to it at a distance of 8 cm from its axis. If now the disc with mud makes 150 rpm only, calculate the moment of inertia of the disc.

11) What is the moment of inertia of a solid cylinder of mass M and radius R about axis tangential to cylinder surface and parallel to the axis of cylinder?

12) If two point masses are placed at (+2 m), and (- 2 m), is it necessary that the centre of mass of system must lie at origin?

13) A fan of moment of inertia is 0.6 kg-m^2 is to be run upto a working speed of 0.5 rps. What is the angular momentum of the fan?

14) Since his childhood Sanjay had always seen his mother grinding flour in the grindstone. He had observed that his mother had to do a lot of hand work in order to get flour from wheat. He felt very helpless at that time. As he grew older he thought of an idea to connect an electric motor to the wheel of grindstone. Now, it become very easy to get flour with help of grindstone and now his mother is very happy and felt proud of his intelligence.
List some qualities of Sanjay.

15) Since his childhood Sanjay had always seen his mother grinding flour in the grindstone. He had observed that his mother had to do a lot of hand work in order to get flour from wheat. He felt very helpless at that time. As he grew older he thought of an idea to connect an electric motor to the wheel of grindstone . Now, it become very easy to get flour with help of grindstone and now his mother is very happy and felt proud of his intelligence.
A grinding stone of diameter 4 m revolving at 120 rpm accelerates to 660 rpm in 9s. Calculate the angular acceleration and linear acceleration.

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When his mother uses grindstone to get flour, what energy transformation takes place?

17) Since his childhood Sanjay had always seen his mother grinding flour in the grindstone. He had observed that his mother had to do a lot of hand work in order to get flour from wheat. He felt very helpless at that time. As he grew older he thought of an idea to connect an electric motor to the wheel of grindstone . Now, it become very easy to get flour with help of grindstone and now his mother is very happy and felt proud of his intelligence.
After attaching motor to grindstone wheel, what energy transformation takes place?

18) Find the centre of mass of a triangular lamina.

19) Does angular momentum of a body in translatory motion is zero?

20) Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

21) Does tile radius of gyration depend on the angular velocity of the body?

22) If a string of a rotating stone breaks, in which direction will the stone move?

23) Which component of a force does not contribute towards torque

24) Two satellites of equal masses are orbiting at different heights. Will their moments of inertia be the same or different?

25) What is moment of inertia of a solid sphere about its diameter?

26) What is moment of inertia of a hollow sphere about an axis passing through its centre?

27) What type of motion is produced by couple?

28) Can a body in translatory motion have angular momentum?

29) Why is the head of screw made wide?

30) What is the direction of the torque of a force?

31) What is rotational analogue of mass of a body?

32) What are the two theorems of moment of inertia?

33) Is radius of gyration of a body a constant quantity?

34) What are the factors on which moment of inertia of a body depend?

35) A labourer standing near the top of an old wooden step-ladder feels unstable. Why?

36) Why does a girl have to lean towards right when carrying a bag in her left hand?

37) When do we call a body rigid?

38) Is it possible to open a pen cap with one finger? Why?

39) Why do we place handles at maximum possible distance from the hinges in a door

40) When the earth shrinks, without reducing its mass, what change will be there in the duration of a day?

41) Name the quantity which can bring rolling without slipping

42) Can centre of mass of a body coincide with geometrical centre of the body?

43) What are the units and dimensions of moment of inertia? Is it a vector quantity

44) State the condition for translational equilibrium of a body.

45) State the condition for rotational equilibrium of a body

46) Should the centre of mass of a body necessarily lie inside the body? Explain

47) What is rotational analogue of force?

48) Raja, a student of class V was playing with 'LATTU' with his elder brother who was studying in XI science. Raja asked his brother, why this lattu is rotating so fast about a sharp nail and did not fall even in tilted position. His elder brother explained Raja about the centre of mass and centre of gravity of the body.
(i) What values are displayed by Raja?
(ii) A constant torque of 1000 Nm turns a wheel of moment of inertia 200 kg m^2 about an axis through its centre. What is its angular velocity after 3 seconds?

49) Deepak went to a fair with his father. There a show was going on. He saw that a person sits near the edge of a circular platform revolving with a uniform angular speed. After sometime the person gets up and starts moving from the edge towards the centre of the platform. Deepak observed that the angular velocity of the moving plateform increases. He wants this answer from his father. His father explained him that when moment of inertia of body decreases, the angular velocity increases.
(i) What are the values of Deepak displayed here?
(ii) What is the relation between angular momentum, moment of inertia and angular velocity.

50) Find the scalar and vector products of two vectors $\mathbf{a} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ and $\mathbf{b} = (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

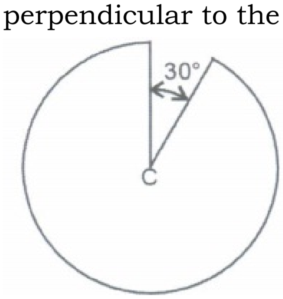
51) In HCI molecule, separation of nuclei of the two atoms is about 1.27Å (1Å = 10^{-10} m). Find the centre of mass of molecule given that a chlorine atom is about 35.5 times massive than a hydrogen atom.

52) If no external force is acting on a two body system, what will happen to
(a) velocity of COM?
(b) angular momentum?

53) Prove the Kepler's law, that the line joining the sun and the planet sweeps equal areas in equal time, using the angular momentum conservation with the planet.

54) The moment of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momentum are equal. Which of them has greater kinetic energy?

55) From a complete ring of mass M and radius R, a 30° sector is removed. What is the moment of inertia of the incomplete ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring?



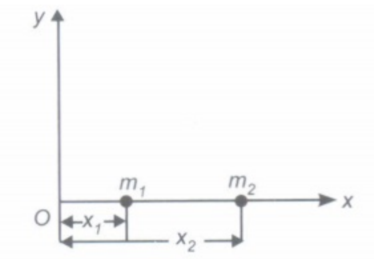
56) A car is moving on road with speed 54 kmh^{-1}. What should be the value of torque if the car is brought to rest in 15 seconds? Radius and moment of inertia of wheel about the axis of rotation are 0.35 m and 3 kgm^3 respectively.

57) Establish the relation between angular momentum and rotational kinetic energy.

58) A disc of mass M. and radius r is rotating with an angular velocity ω. If gently, two masses m are placed at a distance $\frac{r}{2}$ on either side of the axis, what will be its angular velocity?

59) Name the physical quantity corresponding to force in rotational motion. How is it related to force and give its units?

60) What will be the centre of mass of the pair of particles described below in figure on the x-axis?



61) If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved? Explain.

62) Can a body in translatory motion have angular momentum? Explain

63) A ring, a disc and a sphere all of the same radius and same mass roll down on an inclined plane from the same height h. Which of the three reaches the bottom (i) earliest (ii) latest ?

64) State whether the statement given below is true or false giving reason in brief.
A ring of mass 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius are given the same kinetic energy and released simultaneously on a flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from the ring and cylinder. The rolling friction in both the cases is negligible. The cylinder will reach the wall first.

65) Two identical cylinders 'run a race' starting from rest at the top of an inclined plane, one slides without rolling and other rolls without slipping. Assuming that no mechanical energy is dissipated in heat, which one will win?

66) Two solid spheres of the same mass are made of metals of different densities, which of them has larger moment of inertia about its diameter? Why?

67) Find the torque of a force $(7\hat{i}+3\hat{j}-5\hat{k})\mathbf{N}$ the origin, the force acts on a particle whose position vector is $(\hat{i}-\hat{j}+\hat{k})m$.

68) Briefly explain motion of the centre of mass of earthmoon system.

69) Write an expression for the moment of inertia of a circular disc of mass M and radius R
(i) about an axis passing through its centre, and perpendicular to its plane.
(ii) about its diameter.
(iii) about a tangent to its own plane.
(iv) about a tangent perpendicular to the plane of the disc.

70) Write an expression for the moment of inertia of a ring of mass Mand radius r.
(i) about an axis passing through its centre and perpendicular to its plane
(ii) about its diameter.
(iii) about a tangent in its own plane.
(iv) about a tangent perpendicular to the plane of the ring.

71) Name the physical quantity corresponding to inertia in rotational motion. How is it calculated? Give its units.

72) Two bodies of masses 1 kg and 2 kg are located at (1, 2) and (-1, 3) respectively. Calculate the coordinates of the centre of mass.

73) If earth were to shrink suddenly (keeping the mass same), what would happen to the length of the day?

74) What is a couple? What effect does it have on a body? Show that moment of couple is same irrespective of the point of rotation of the body.

75) An electron of mass 9×10^{-31} kg revolves in a circle of radius 0 53 Å around the nucleus of hydrogen with a velocity of 2.2×10^{6} ms^{-1}. Show that angular momentum of electron is $h/2\pi$, where h is Planck's constant.

76) What is torque? Write its formula in vector form. Handle, to open the door, is always provided at the free edge of a door. Why?

77) . Derive the relations
(i) $L = I\omega$ (ii) $\tau = I\alpha$

78) Torques of equal magnitude are applied to hollow cylinder and a solid sphere both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed?

79) Two circular discs A and B of the same mass and same thickness are made of two different metals whose densities are d_A and dB ($d_A > d_B$). Their moments of inertia about the axes passing

through their centres of gravity and perpendicular to their planes are I_A and I_B. Which is greater I_A or I_B ?

80) Find

(i) the radius of gyration and

(ii) the moment of inertia of a rod of mass 100 g and length 100 cm about an axis passing through its centre and perpendicular to its length.

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80 x 2 = 160

1)

When the person withdraws his chest, his moment of inertia decreases. No external torque is acting on the system. So, to conserve angular momentum, the angular velocity increases.

2)

All mass of a hollow cylinder at a distance R from axis of rotation. Whereas in case of a sphere, most of mass lies at a distance less than R from axis of rotation. As moment of inertia is $\sum M_i R_i^2$, so sphere as a lower value of moment of inertia.

3)

The moment of inertia of the system (circular table + two boys) decreases. To conserve angular momentum ($L = I\omega = constant$), the speed of rotation of the circular table increases.

4)

Molten ice from poles into ocean and so mass is going away from axis of rotation. So, moment of inertia of earth increases and to conserve angular momentum, angular velocity (ω) decreases. So, time period of rotation increases ($T = 2\pi/\omega$). So, net effect of global warming is increasing in the duration of day.

5)

In a whirl wind, the air from nearby region gets concentrated in a small space thereby decreasing the value moment of inertia considerably. Since $I\omega$= constant, due to decrease in moment of inertia, the angular speed becomes quite high.

6)

Centre of mass and centre of gravity are two different concepts. But if g goes not vary from one part of body to other than CG and CM coincides.

So, when vertical height of the object is very small compared to a radius of earth, we call object small, otherwise, we call it extended. In above context, building and pond are small objects and a deep lake and a mountain are large extend objects.

7)

M = 20 Kg, $\omega = 100 rad/s, R = 0.25m$.

Moment of inertia of cylinder about its own axis

$= \frac{1}{2}MR^2 = \frac{1}{2} \times 20 \times (0.25)^2$

Rotational KE $= \frac{1}{2}I\omega^2$

$= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.625 \times (100)^2 = 3125J$.

8)

The moment of inertia $\left(I = \frac{1}{2}MR^2\right)$ of the earth about its own axis will become one-fourth and so its angular velocity will become four times$(L = I\omega = constant)$. Hence, the time period will reduce to one-fourth $(T = 2\pi/\omega)$, i.e. 6 hours.

9)

When a cat falls to ground from a height, it stretches its body alongwith the tail so that its moment of inertia become high. Since,$I\omega$ is to remain constant, the value of angular speed ω decreases and therefore the cat is able to land on the ground gently.

10) $I = 3.2 \times 10^{-8} kg - m^2$.

11) $2MR^2$.

12) No

13) 1.9 km-m/s

14) caring, thoughtful, intelligent, creative are some qualities we can observe about Sanjay from this paragraph.

15) Given, $radius, r = \frac{diameter}{2} = \frac{4}{2} = 2m$

$n_1 = 120rpm = \frac{120}{60} rps = 2rps$

$n_1 = 660rpm = \frac{660}{60} rps = 11rps$

$\therefore\ Angular\ acceleration, \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t}$

$= \frac{2\pi(11-2)}{9} = 2\pi rad/s^2$

Linear acceleration, $\alpha = r\alpha = 2 \times 2\pi = 4\pi m/s^2$.

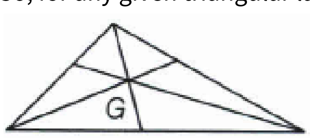
16) Muscular energy to mechanical energy.

17) Electrical energy to mechanical energy.

18) For any planar solid, centre of mass always lies at its geometrical centre.

Geometrical centre of a triangle is intersection point of its media.

So, for any given triangular lamina



Centre of mass is at its centroid, point of intersection is media.

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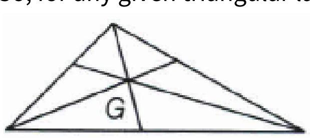
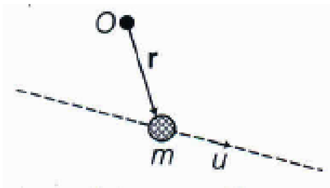


Centre of mass is at its centroid, point of intersection is media.

19) Angular momentum of a body is measured with respect to certain origin.



So, a body in translatory motion can have angular momentum.
It will be zero, if origin lies on the line of motion of particle.

20)

In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.
No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

21) No, $K = \sqrt{\frac{1}{M}}$

22) The stone will move along the tangent at the point of breaking.

23) The radial component of a force does not contribute towards torque.

24) The moments of inertia will be different on account of different distances.

25) I = $\frac{2}{5}$MR^2 where M is the mass and R is radius of the solid sphere.

26) I = $\frac{2}{3}$MR^2 where M is the mass and R is radius of the hollow sphere.

27) Only rotational motion

28)

Yes, a particle in translatory motion always has an angular momentum, unless the point (about which angular momentum is calculated) lies on the line of motion.

29) The head of a screw is made wide so as to have a greater torque for the applied force.

30) Direction of torque of a force is a direction perpendicular to the plane of rotation given by $\vec{\tau} = \vec{r} \times \vec{F}$

31) Rotational analogue of mass is moment of inertia

32) Two theorems of moment of inertia are theorem of parallel axes and theorem of perpendicular axes.

33)

Radius of gyration of a given body depends upon the choice of rotation axis. However, for a given axis of rotation, the radius of gyration of a body has a fixed value.

34) Moment of inertia of a body depends upon

(i) mass of the body

(ii) shape and size of body

(iii) position of the axis of rotation, and

(iv) distribution of mass in the body about the axis of rotation

35)

The point of contact of the ladder with the ground is the point about which the ladder can rotate. When the labourer is at the top of the ladder, the lever arm of force i large. So, the turning effect can. be large.

36)

When the girl carries a bag in her left hand, the centre of gravity of the system is shifted to the left. In order to bring it in the middle, the girl has to lean towards right.

37) When the separation between any two masses consistiting does not vary, the body is said to be rigid.

38) No, since torque cannot be applied.

39) To develop torque with less force being applied.

40) L is conserved. If the earth shrinks, duration of the day decreases.

41) Friction with the surface.

42) when the body has a uniform mass density.

43) The units of moment of inertia are kg m^2 and its dimensional formula is $[M^1L^2T^0]$.No, it is not a vector quantity.

44)

For translational equilibrium of a body net force acting on it i.e., the vector sum of all the forces acting on the body must be zero.

45)

For rotational equilibrium of a body the vector sum of torques of all the forces acting on the body about the reference point must be zero.

46) No, it may lie outside the body. In case of semicircular ring, it is at the centre which is outside the ring.

47) Rotational analogue of force is torque

48) (i) The value displayed are: curiosity, wants to learn something, sharp mind and sensible.

(ii) Angular acceleration $\alpha = \frac{\tau}{I} = \frac{1000}{200} = 5rads^{-1}$

$\therefore$ Angular velocity after 3 sec

$\omega = \omega_0 + \alpha t$

= 0 + 5 x 3

= 15 rad s^{-l}

49) (i) Values are : Sharp mind, keen observation, curiosity and intelligence.

(ii) If L= Angular momentum

I = Moment of Inertia

ω = Angular velocity

$\therefore$ L = Iω

50) $\mathbf{a} \cdot \mathbf{b} = (3\bar{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\overline{\mathbf{k}}) \cdot (-2\vec{\mathbf{i}} + \bar{\mathbf{j}} - 3\overline{\mathbf{k}})$

= -6 -4 -15

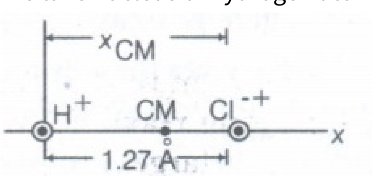
= -25

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \mathbf{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix} = 7\hat{\mathbf{i}} - \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Note $\mathbf{b} \times \mathbf{a} = -7\hat{\mathbf{i}} + \bar{\mathbf{j}} + 5\hat{\mathbf{k}}$

51) Mass of an atom is concentrated at its nucleus, they can be treated as point masses.

We take nucleus of hydrogen atom at origin and nucleus of chlorine atom on x-axis.



where, m_1 = mass of hydrogen = 1

m_2 = mass of chlorine = 35.5

X_1 = 0, x_2 =1.27 A

As the system is symmetrical about x-axis, its centre of mass lies on x-axis.

Using $x_{\text{CM}} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{1\times0+35.5\times1.27}{1+35.5}$

= 1.235 Å

= 1.235 x 10^{-10} m(from origin)

So, centre of mass of HCI molecule is nearly 1.235 Å from the H-nucleus on the line joining H and CI nuclei.

52) (a) Velocity of centre of mass will be same.

(b) Angular momentum is conserved and will be zero, if no rotational motion exists.

53)

When the planet moves along the line joining the sun and the planet it sweeps some area given by

$A = \frac{1}{2}r^2\theta$ where θ is the angular displacement.

$\therefore \quad \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$

$\frac{dA}{dt} = \frac{1}{2m}mr^2\omega = \frac{L}{2m}$

Since no torque acts, angular momentum Lis a constant, so $\frac{dA}{dt}$ is a constant, i.e., the line Joining t e sun and the planet sweeps equal areas in equal intervals of time.

54) We know that angular momentum, L = Iω and K.E. of rotation,

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2I}(L^2)$

When L is constant, then

$K \propto \frac{1}{I}$

$\because I_A > I_B$

ஃ $K_A < K_B$ or $K_B > K_A$

i.e. the body B has greater K.E. of rotation than the body A.

55) Mass of Incomplete rIng $= M - \frac{M}{2\pi} \times \frac{\pi}{6}$

$= M - \frac{M}{12} = \frac{11}{12}M$

Moment of inertia of incomplete ring

$= \left(\frac{11M}{12}\right)R^2 = \frac{11}{12}MR^2$

56) Speed of car = 54 kmh^{-1} = 15 ms^{-1}

Radius of a car = 0.35 m

Angu ar velocity $\omega 0 = \frac{\text{Speed}}{R} = \frac{15}{0.35}$, Initial $\omega_t = 0$

Angular acceleration $\alpha = \frac{\omega_t - \omega_0}{t}$

$= \frac{0 - \frac{15}{0.35}}{15} = \frac{-1}{0.35}\text{rads}^{-2}$

Torque $\tau = I\alpha = 3 \times \left(\frac{-1}{0.35}\right)\text{kgm}^2\ \text{s}^{-2} = -8.57\ \text{kg m}^2\ \text{s}^{-2}$

57) Angular momentum L = I ω

Rotational K.E.,E_k= I ω^2

$E_k = \frac{1}{2}\frac{(I\omega)^2}{I} = \frac{L^2}{2I}$

58) Angular momentum initially

$= \frac{Mr^2}{2} \cdot \omega$ [.: L=Iω]

When the masses are placed,

$I = \frac{Mr^2}{2} + \left[m\left(\frac{r}{2}\right)^2\right] \times 2$

$= \frac{r^2}{2}[M + m]$

As I increases, ω decreases because angular momentum is to be conserved.

New angular velocity, ω'

$= \frac{\frac{Mr^2}{2}\omega}{\frac{r^2}{2}[M+m]} = \frac{M\omega}{(M+m)}$

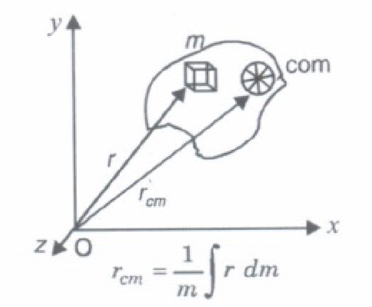
59) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

S.l. Unit = Nm

60)

The centre of mass of the pair of particles is located on the x-axis and lies somewhere between the particles. The x-coordinate of the centre of mass in this case would be,

$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$



61)

No, rotational KE is not conserved, as

$\mathrm{I}_1\omega_1 = \mathrm{I}_2\omega_2$

$\mathrm{I}_1^2\omega_1^2 = \mathrm{I}_2^2\omega_2^2$

$\mathrm{I}_1\left(\frac{1}{2}\mathrm{I}_1\omega_1^2\right) = \mathrm{I}_2\left(\frac{1}{2}\mathrm{I}_2\omega_2^2\right)$

$I_1K_1 = 1_2K_2$

If $I_2 < I_1$ then $K_2 > K_1$

i.e. when MI decreases, rotational KE increases.

62)

Yes, a body in translatory motion shall have angular momentum unless the fixed point, about which angular momentum is taken, lies on the line of motion of the body. This follows from $|\mathrm{L}| = rp\sin\phi$ L = 0, only when $\phi = 0 \text{ or } \phi = 180^\circ$.

63)

We have already deduced that acceleration of an object down on inclined plane is given by

$a = \frac{g\sin\theta}{1+(\mathrm{I}/\mathrm{m}r^2)}$ ....(i)

For a ring, I = mr2

$\therefore\ a_{\text{ring}} = \frac{g\sin\theta}{1+1} = 0.5g\sin\theta$

For a disc $\mathrm{I} = \frac{1}{2}mr^2$

$\therefore \quad a_{\text{disc}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$

= 0.67 g sin 8

For a sphere $\mathrm{I} = \frac{2}{5}mr^2$

$\therefore \quad a_{\text{sphere}} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta = 0.71g\sin\theta$

As a_{sphere} is maximum, it will reach the bottom at the earliest. Again as a_{ring} is minimum, it will reach the bottom at the end.

64)

In case of rolling, total kinetic energy

$\mathrm{K}=\frac{1}{2}mv^2+\frac{1}{2}\mathrm{I}\omega^2$

with ω = v/r

$\mathrm{K}=\frac{1}{2}mv^2+\frac{1}{2}\mathrm{I}\frac{v^2}{r^2}=\frac{1}{2}$

$=mv^2\left[1+\frac{\mathrm{I}}{mr^2}\right]$

For a ring, 1 = mr2

$\therefore\ \mathrm{K}_r=\frac{1}{2}mv^2(1+1)=mv^2$

$v=\sqrt{\frac{\mathrm{K}_r}{m}}=\sqrt{\frac{\mathrm{K}_r}{0.3}}$

For a cylinder $\mathrm{I}=\frac{1}{2}mr^2$

$\therefore\ \mathrm{K}_c=\frac{3}{4}mv^2$

$v=\sqrt{\frac{4\,\mathrm{K}_c}{3m}}=\sqrt{\frac{\mathrm{K}_c}{\frac{3}{4}\times 0.4}}$

$=\sqrt{\frac{K_c}{0.3}}$

As $K_r = K_c$, therefore, velocity (v) of both the ring and cylinder is same. As the motion is uniform, both the ring and cylinder will reach the wall at the same time. The given statement is false.

65) When a cylinder slides without rolling

$\mathrm{E}=\frac{1}{2}mv_1^2, v_1=\sqrt{2\mathrm{E}/\mathrm{m}}$

When the cylinder rolls without slipping

$\mathrm{E}=\frac{1}{2}mv^2+\frac{1}{2}\mathrm{I}\omega^2$

$\mathrm{E}=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$

$=\frac{1}{2}mv^2+\frac{1}{4}mv^2 \qquad [\because v=r\omega]$

$=\frac{3}{4}mv^2$

$v=\sqrt{\frac{4\mathrm{E}}{3m}}$

As $v_1 > v$, therefore sliding cylinder will win the race.

66) Let mass .of two spheres (A and B) be M and radius of two spheres be r_A and r_B.

Then $\frac{4}{3}\pi r_{\mathrm{A}}^3 d_{\mathrm{A}}=\frac{4}{3}\pi r_B^3 d_B$

where d_A and d_B are densities of two spheres respectively.

$\frac{r_A^3}{r_B^3}=\frac{d_B}{d_A}$

$\frac{I_A}{I_B}=\frac{\frac{2}{5}Mr_A^2}{\frac{2}{5}Mr_B^2}$

$=\left(\frac{d_B}{d_A}\right)^{\frac{2}{3}}$ as $\frac{r_{\mathrm{A}}}{r_{\mathrm{B}}}=\left(\frac{d_{\mathrm{B}}}{d_{\mathrm{A}}}\right)^3=\left(\frac{d_B}{d_A}\right)^3$

If $d_A > d_B$ then $I_B > I_A$ ·

Moment of inertia of sphere having lesser density will be more. Therefore, sphere B has larger moment of inertia.

67) $F=7\hat{i}+3\hat{j}-5\hat{k}\,\mathrm{N}$

Position vector $\vec{r}=\hat{i}-\hat{j}+\hat{k}\,\mathrm{m}$

The torque or moment of a force is given by

$\vec{\tau}=\vec{r}\times\vec{F}$

$$\vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 7 & 3 & -5\end{vmatrix}$$

$\vec{\tau}=\hat{i}(5-3)-\hat{j}(-5-7)+\hat{k}(3+7)$

$\vec{\tau}=2\hat{i}+12\hat{j}+10\hat{k}$

68)

The centre of mass of a earth-moon system moves in elliptical path around the sun. The system of earth-moon moves due to the external force provided by the sun, and the parts of the system under motion about COM is due to the internal force between them.

69) (i) $\mathrm{I}=\frac{\mathrm{MR}^2}{2}$

(ii) $\mathrm{I}_d=\frac{\mathrm{MR}^2}{4}$

(iii) $\mathrm{I}_{tp}=\frac{\mathrm{MR}^2}{4}+\mathrm{MR}^2=\frac{5}{4}\mathrm{MR}^2$

(iv) $\mathrm{I}_{t\perp}=\frac{\mathrm{MR}^2}{2}+\mathrm{MR}^2=\frac{3}{2}\mathrm{MR}^2$

70) (i) MR^2

(ii) $\frac{\mathrm{MR}^2}{2}$

(iii) $\frac{\mathrm{MR}^2}{2}+\mathrm{MR}^2=\frac{3}{2}\mathrm{MR}^2$

(iv) $MR^2 + MR^2 = 2\ MR^2$

71) Moment of Inertia (I)

1 = MK^2 , where K $=\sqrt{\frac{r_1^2+r_2^2\ldots r_n^2}{n}}$

where r_1, r_2 , r_3 , are all perpendicular distances from the axis of rotation. Its S.I. unit is kg m^2.

72) $\mathrm{X}_{\mathrm{cm}}=\frac{m_1x_1+m_2x_2}{m_1+m_2}$

$=\frac{1\times1+2\times(-1)}{3}=\frac{-1}{3}$

$\mathrm{Y}_{\mathrm{cm}}=\frac{m_1y_1+m_2y_2}{m_1+m_2}=\frac{1\times2+2\times3}{3}$

$=\frac{9}{3}=\frac{8}{3}$

Coordinate of centre of mass (COM) is $\left(\frac{-1}{3},\frac{8}{3}\right)$

73) Keeping the mass same if the earth shrinks, the angular momentum will remain constant.

$\therefore\ (\mathrm{I}\omega)_{\mathrm{B}}=(\mathrm{I}\omega)_{\mathrm{A}}$

i.e., $\frac{2}{5}\mathrm{MR}^2\cdot\frac{2\pi}{24}=\left(\frac{2}{5}\mathrm{M}\cdot r^2\right)\frac{2\pi}{\mathrm{T}}$

i.e., $\mathrm{T}=\frac{r^2}{\mathrm{R}^2}24$

Since r < R, T< 24 hours.

74)

Couple: When equal parallel forces act on the body in opposite direction at two different points, they form a couple. The net force due to couple is zero, but they exert a torque and produce rotational motion. Let us consider a couple acting on a rigid body at a point A and B with position vectors r_1 and r_2 with respect to origin $O. \vec{F}$ and $-\vec{F}$ are forces acting on A and B respectively.

The moment of couple = moment of force at A + moment of force at B

$=\vec{r}_1 \times(-\vec{F})+\vec{r}_2 \times \vec{F}=(\vec{r}_2-\vec{r}_1) \times \vec{F}$

But $\vec{r}_1+\overrightarrow{AB}=\vec{r}_2$

$\Rightarrow \quad \overrightarrow{AB}=\vec{r}_2-\vec{r}_1$

∴ moment of couple = $=\overrightarrow{AB} \times \vec{F}$ which shows it is independent of point of rotation.

75) Here, m = 9 x 10^{-31} kg

r = 0.53 Å

= 0.53 x 10^{-10} m

v = 2.2 x 10^{6} m/s

Angular momentum,

L = mvr

= 9 x 10^{-31} x 2.2 x 10^{6} x 0.53 x 10^{-10}

L = 1.0494 x 10^{-34} Js

Also $\frac{h}{2\pi}=\frac{6.6\times10^{-34}}{2\times22/7}=1.05\times10^{-34}$Js

Hence, L = h/2π

76)

Torque is the moment of force. It is written as the cross-product of the position vector of the point of application of the force and the force, i.e

$\vec{\tau}=\vec{r}\times\vec{F}$

It is measured in Nm and has dimensions of ML^2T^{-2} .Since $\tau=r\,F\sin\theta$ where θ is the angle between $\vec{r}$ and $\vec{F}$, it is also said, torque is the product of the perpendicular distance between the axis of rotation and point of application of force in line with the force. It is the cause of rotational motion.

77) (i) To prove L = Iω

We know, L = r x p = r x mv

$=rmr\omega=mr^2\omega=I\omega$

(ii) To prove τ = Iα

We know τ = r F = rma

$mr^2\alpha = I\alpha$

78)

MI of solid sphere about an axis through its centre, $I_2=\frac{2}{5}MR^2$ and

I_1 = MI of cylinder = MR^2

Torque applied $\tau=I_1\alpha_1=I_2\alpha_2$

$\therefore \frac{\alpha_2}{\alpha_1}=\frac{I_1}{I_2}=\frac{MR^2}{\frac{2}{5}MR^2}=\frac{5}{2}$

$\therefore \alpha_2>\alpha_1$

From $\omega=\omega_0+\alpha t$, we find that for given ω_0 and t, $\omega_2>\omega_1$ i.e., angular speed of solid sphere will be greater than the angular speed of hollow cylinder.

79) Let r_A and r_B be the radii of discs A and B. As their mass (m) and thickness (t) are same, therefore,

$m = (\pi r_A^2)\, t \times d_A = (\pi r_B^2) \times t \times d_B$

$\therefore \quad \frac{r_A^2}{r_B^2} = \frac{d_B}{d_A}$

Now $\frac{I_A}{I_B} = \frac{\frac{1}{2} m r_A^2}{\frac{1}{2} m r_B^2}$, but as $\frac{r_A^2}{r_B^2} = \frac{d_B}{d_A}$

and $d_A > d_B, \quad \therefore I_B > I_A$

80) Moment of inertia

$I = \frac{Ml^2}{12} = \frac{0.1 \times 1}{12} = 0.008 \text{kgm}^2$

If radius of gyration is K then, $I = MK^2$

$\Rightarrow \quad K = \frac{l}{\sqrt{12}} = \frac{1}{\sqrt{12}} = 0.0289 \text{ m}$